How to Model Intermodulation Distortion

Stephen A. Maas

Dept. of Electrical Engineering, University of California, Los Angeles

Abstract: This paper examines the problem of calculating intermodulation levels and mixer spurious responses in nonlinear microwave circuits. We examine the effects of device models, analytical methods, dynamic range, and linear elements on accuracy. We also describe ways to obtain accurate analyses of these phenomena using available harmonic-balance and Volterra-series circuit simulators.

I. Introduction

In recent years the availability of general-purpose harmonic-balance and Volterra-series simulators has made it possible for engineers to convince themselves that they can calculate intermodulation (IM) levels of a wide variety of microwave components. However, this conviction has been more of a religious belief than a scientific one: for reasons not always clear to the user, these analytical tools often fail to predict IM levels accurately. In other cases, the huge amount of computation required to obtain a meaningful result makes such calculations appear to be impractical.

Fortunately, with a little understanding one can circumvent many (but, alas, not all) of these problems. The difficulties usually reside in one or more of several critical areas; these are (1) the device model; (2) the method of analysis; (3) the dynamic range of the calculation; (4) the selection of frequency components; and (5) limitations of the linear-circuit models. Unfortunately, not all of these factors are under the user's control.

II. What Affects the Accuracy of IM Calculations?

It may be surprising to some engineers that, in spite of the intense work in the last few years on the simulation of nonlinear microwave circuits, very little work has been done to answer the following deceptively simple question:

What properties of a device model or an analytical technique affect the accuracy of a nonlinear circuit analysis?

The answer is usually assumed without proof: use as many frequency components as possible (within, of course, practical limits), make sure that the model accurately reproduces the device's I/V and Q/V characteristics, and model the linear part of the circuit as well as possible. These assumptions represent only about half the answer. We will address the rest of the answer in the rest of this paper.

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A. Device models

The earliest attempts at what eventually became known as harmonic-balance analysis [1-5] were applied to diode mixers. Fortunately for those researchers, the Schottky-barrier diode is a solid-state device (perhaps the only device) that can be modeled adequately, for almost all purposes, by a simple set of closed-form, quasi-static equations. The success of these efforts (and they were indeed successful) focussed subsequent work primarily on analytical techniques, and less on the modeling of solid-state devices. As FET technology matured, it was assumed that the same techniques that worked for diodes would work for FETs as well: characterize the I/V characteristics of the dominant resistive nonlinearities and the C/V or Q/V characteristics of the reactive nonlinearities. As it happened, this approach was adequate for many FET applications: for fundamental-frequency circuits such as power amplifiers, for low-harmonic frequency multipliers, and for mixers using large-signal/small-signal analysis [6]. However, engineers eventually recognized that it was not working when they tried to apply it to intermodulation analysis.

Why?

This approach is flawed by the fact that it models only the static nonlinearities in the device. If these are well modeled, first-order nonlinearities--which are proportional to the derivative of the static nonlinearities--usually are adequately modeled as well. However, higher order nonlinearities are not automatically well modeled, especially in weakly nonlinear devices such as FETs. The reason, most fundamentally, is that modeling a real function accurately does not automatically model its derivatives accurately.

It is possible to show that the levels of nth-order intermodulation components depend most strongly on the nth and lower derivatives of the dominant I/V or Q/V nonlinearities. Thus, if you want to obtain good results for third-order IM calculations, your device model must accurately express the first three derivatives of the device's I/V characteristic; this generally will not be the

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case if you take care to model only the static I/V. With this point in mind, we can formulate our first rule:

The First Law of IM Analysis: *To achieve accurate nth-order IM calculations, one must model accurately the device's I/V or Q/V characteristic and its first n derivatives.*

It is important to note that this stipulation is independent of the means used to model the device; it is not limited to closed-form expressions. Thus, even if the device is modeled by a fairly abstruse numerical calculation, the derivatives of the I/V and Q/V characteristics implicit in that model must be correct for the calculation to be accurate. Also, it is valid regardless of the type of analysis employed (more on that below).

Schottky diodes are nearly ideal exponential devices, and the standard methods for modeling them [6] work well for IM analysis. In FETs, however, the situation is more complicated. In FETs the controlled drain-current source is the dominant nonlinearity, and many of the conventional techniques for modeling a FET do not automatically express its derivatives adequately. Methods for measuring and modeling the derivatives of this nonlinearity have been developed [7-8]; one effective method is to extract the derivatives from measurements of low-frequency small-signal harmonics, much as one might use gain measurements to find the FET's transconductance. It is usually impossible to obtain the derivatives from dc measurements; the derivatives are small, and are lost in measurement error.

A second consideration should be obvious, but it is ignored often enough to justify stating it formally:

The Second Law of IM Analysis: *to achieve accurate calculations of any kind (linear or nonlinear), one need model the device only over the range of voltages or currents it experiences in operation.*

If the FET is used as a small-signal amplifier, the voltages across its nonlinear elements deviate only incrementally from their bias values. Therefore, it is necessary to know only the I/V derivatives at the gate-bias voltage. However, in FET mixers a large-signal LO voltage is applied to the gate, so the gate voltage varies continuously between pinchoff and a small forward-bias voltage. In this case, it is necessary that the model express the derivatives accurately over this *entire* range. This makes the modeling of FETs for IM analysis of mixers considerably more demanding than for amplifiers.

B. The method of analysis

With today's heavy emphasis on harmonic-balance analysis, it is easy to forget that harmonic balance is only one of several ways to analyze nonlinear circuits. Time-domain methods (embodied in programs such as SPICE II) have a place in the microwave world (although SPICE is not well suited for IM analysis), and Volterra-series methods have advantages that complement harmonic balance. Volterra-series analysis avoids many of the problems inherent in harmonic balance, and is the method of choice for calculating levels of small-signal IM and related phenomena (e.g. AM/PM conversion and desensitization).

The main advantages of Volterra-series analysis over harmonic balance are that it is noniterative, requires less computer memory, and does not require Fourier transforms; thus, its computation cost is lower and dynamic range is far greater. Furthermore, one need not develop a mathematical expression to model the device's I/V characteristic; all that is necessary are the Taylor-series coefficients of the nonlinearities' I/V characteristics (i.e., the derivatives) at their bias voltages. The disadvantage of Volterra methods is that they are limited to small excitations and weak nonlinearities. Nevertheless, the complementarity between harmonic balance and Volterra analysis is very gratifying: each succeeds primarily where the other fails. Thus, one more law:

The Third Law of IM Analysis: *Use the appropriate method of analysis. There are choices, you know.*

C. Dynamic range of the calculation

It may surprise some to hear that numerical calculations have a dynamic range: the ratio of the largest to the smallest meaningful numbers is limited. The "numerical noise" that defines the bottom end of this range comes mostly from mathematical operations where the accuracy of the result is unusually sensitive to errors in the input data. Although classical fast Fourier transforms (FFTs), used with single-tone excitation, are remarkably good in this respect, the discrete Fourier transforms (DFTs) used in most harmonic-balance software, with multiple noncommensurate excitation frequencies, are much poorer. The result is that the numerical "trash" generated by the use of such transforms may be only a few tens of dB below the largest signals, and often is much larger than the IM products themselves. The IM levels that result from a calculation in such a situation are clearly meaningless. This dynamic-range limitation of harmonic balance is one of the best reasons for using Volterra methods for small-signal IM calculations.

These dynamic-range limitations are not as easy to circumvent as they might at first appear. The obvious solution is to use larger excitation levels in the analysis; as long as the levels are kept well below the compression level of the device, accurate intercept-point calculations should be possible. However, for many devices that have good linearity but low compression levels (e.g., heterojunction bipolar transistors), the signal levels must fit in a narrow window between compression and numerical oblivion. Put simply, the dynamic range of the calculation is too low to be practical.

This gives us our fourth law:

The Fourth Law of IM Analysis: *Be sure you are aware of the dynamic-range limitations of the analytical method you are using.*

D. Selection of frequency components

The conventional wisdom states (in part, correctly) that a nonlinear circuit, excited by the frequencies f_1, f_2, f_3 ... generates the set of mixing frequencies $m\,f_1 + n\,f_2 + p\,f_3 + \ldots$ where m, n, p ... are integers. However, IM components at many of these frequencies are insignificant; conversely, limiting the set to some maximum value of $m + n + p + \ldots$, the usual practice, often omits important frequencies. When the excitations are small, Volterra analysis can be used to identify the significant frequencies [7].

Unfortunately, very few commercially available harmonic-balance simulators give the user the freedom to select for himself the set of frequencies to be used in an analysis. His only option in this case is to use a large set of IM frequencies, and to be prepared to wait a long time for a solution to emerge from the computer. This situation is unfortunate, because the use of the minimal set of frequencies, instead of the full set, could increase the efficiency of such simulators significantly. This practice need not conflict with the need for a larger set of frequencies in the DFT to achieve adequate dynamic range: the set of frequencies used for balancing can be a subset of that used in the DFT, and the sampling methods used in the DFT can be quite independent of the selection of frequencies. Thus, our fifth law:

The Fifth Law of IM Analysis: *If possible, use only the minimal set of frequencies necessary in a harmonic-balance analysis.*

E. Linear-element models

Many of the significant mixing frequencies that are generated by a nonlinear component occur at remarkably high or low frequencies. Currents and voltages at these frequencies appear not only in the nonlinear part of the circuit, but also in the linear part. Thus, the models of such structures as microstrip discontinuities may have a significant effect on the accuracy of the calculation.

Unfortunately, it is rarely possible to model elements such as microstrip discontinuities at very high frequencies. This is less of a problem than it might appear to be, because the parasitic capacitances of the solid-state device very probably will short-circuit it at high frequencies, limiting the effect of inaccuracies in the models of linear-circuit elements. In spite of this, it is important that the accuracy of linear-element models degrade gracefully at both high and low frequencies. Often such models are untested at high frequencies, and their behavior is decidedly bizarre outside their range of interest. For example, the author has encountered one microstrip discontinuity model that created a dc voltage in the junction; this effect was not noticed when it was used in a linear, RF-frequency circuit simulator, but in a harmonic-balance simulator it offset the dc bias and thus presented a serious problem. Again, this phenomenon is not under the user's control.

The Sixth Law of IM Analysis: *Be aware of possible limitations of circuit models at unusually low or high frequencies, and expect surprises.*

III. Conclusion

This paper has discussed some of the factors affecting the accuracy of intermodulation calculations. In order to perform such calculations successfully, the expression for the I/V characteristic of the solid-state device must accurately reproduce not only the characteristic itself, but also its derivatives. One must furthermore choose the most appropriate analytical technique and be mindful of the dynamic-range limitations of Fourier transforms, the need to use an adequate and correct set of frequency components, and finally of the limitations of linear-element models.

IV. References

1. S. Egami, "Nonlinear, Linear Analysis and Computer-Aided Design of Resistive Mixers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, no. 3, 1974, p. 270.

2. A. R. Kerr, "A Technique for Determining the Local Oscillator Waveforms in a Microwave Mixer," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, no. 10, 1975, p. 828.

3. D. N. Held and A. R. Kerr, "Conversion Loss and Noise of Microwave and Millimeter-Wave Mixers: Part 1-Theory and Part 2--Experiment," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, no. 2, 1978, p. 49.

4. S. A. Maas, *Microwave Mixers*, Artech House, Norwood, MA, 1986.

5. B. Schuppert, "A Fast and Reliable Method for Computer Analysis of Microwave Mixers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, no. 1, 1986, p. 110.

6. S. A. Maas, *Nonlinear Microwave Circuits*, Artech House, Norwood, MA, 1988.

7. S. A. Maas and D. Neilson, "Modeling MESFETs for Intermodulation Analysis of Mixers and Amplifiers", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-38, no. 12, 1990, p. 1964.

8. S. A. Maas and D. Neilson, "Modeling GaAs MESFETs for Intermodulation Analysis" *Microwave J.*, May, 1991.